CULTURAL VALUE OF MATHEMATICS

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"The intellect never slumbers," but is ever searching for knowledge and truth. It is ever groping about in the darkness of error and of doubt, and if truly honest in its search, uses every light which an all wise and loving Master has given it to detect the slightest flaw in every finite problem which is presented to it. Its restlessness continues until the goal of absolute certitude is reached.

Perhaps an alluring invitation urges that intellect to turn aside into a by-path easier and more attractive than the rough hard road of truth, which appeals only to him who is morally and intellectually rugged. If so, its faithful friend philosophy calls out to it reminding it that fact is the beginning and end of all things, that fact will not, and cannot be brushed aside. That whether in a purely intellectual pursuit or in the stern realities of an every-day common-place struggle for existence, "Truth is mighty and must prevail."

It reminds that intellect that to every manifestation of effort or action in a substance there is a proper proportioned object on account of which this power is put forth in its specific activity. That effect is always consequent upon a cause, and that that intellect seeking enlightenment must first of all, recognize itself as a created thing, and an effect of a great cause which lives beyond the finite, and without which nothing could be made that was made.

The human intellect made therefore for truth is tireless in its search for knowledge. This is its instinctive quality and is as constant and unvarying as the daily rotation of the earth in its annual journey around the sun. "The heart was made for God," says St. Augustine, "and it cannot rest until it rests in God." As God is truth, this restless struggle of the intellect against error is but the carrying out of the divine law which reminds him who seeks truth that he shall find his Master.

Traveling in the foot-prints of philosophy is education, another of the hand-maids of the intellect. "Could it be made perfect it would fill the world with truth, goodness and beauty, which are the substance of ideals and ever lure the noblest souls to heroic striving and enduring." (Spalding.)

Mathematics is one branch of education which by its very nature seeks this form of perfection. It treats not of the approximate or problematic, but of the exact measuring of quantities or magnitudes, of the exact ascertainment of their properties and relations, and of unvarying methods by which, in accordance with these relations, quantities sought are deducable from others, either known or supposed. It is the science that deals with absolute truth, having for its function to develop the consequences involved in the definition of a group of a mathematical conception, which conceptions in turn have been definitely and absolutely determined by a fixed number of specifications.

To use a familiar example 2 plus 2 makes 4. No matter where we may be, no matter what the application, or what changes of circumstance, of surroundings, or of conditions may be operating, experience has taught us that no exception exists but that 2 added to 2 always makes 4.

This simple mathematical axiom, typical of the science in its most intricate as well as its least complicated workings, appeals to the tender little child counting his shells and pebbles on the shining sands that line the sea, his untutored mind gradually developing from the darkness of an all but animal ignorance and awakening to the fact of numbers. It appeals with equal strength to the magnate of uncounted wealth, sitting gray and feeble in the chimney-corner of life, the shadows growing longer and longer behind him as his palsied hand still toys with his shells and pebbles, now grown into dollars and cents, into dividends and interest. It is the diapason of the musician, producing harmony, universal concord, correct pitch, in his compositions. It is the square and compass of the master-builder dropping for him his plummet, directing his circles, and calling in his angles in order that a magnificent piece of architecture may rear its head above the earth to glorify God through the handiwork of one of his creatures. It guides the eye and directs the nimble fingers of the designer as the intricate mosaic shapes itself with kaleidoscopic beauty at the command of his genius. It stands by the side of the chemist, watching the beam of his balance, jealously counting each miniim and each grain, as he strives to wrest some secret which jealous nature still guards,

locks within her bosom. It accompanies the physicist to his laboratory, the biologist to his microscope, the astronomer on his journey to the uncharted heaven and ever and always it cries out "Truth is mighty and will prevail," "2 plus 2 makes 4."

In the crypt of St. Paul's in London is the tomb of Sir Christopher Wren, its architect. On the wall above is a tablet with the inscription containing the celebrated words, *"Lector si monumentum requires—circumspice."* Well indeed may one look at the cultured world and proclaim it a testimonial to that science, which, both in its abstract consideration and its utilitarian applications, has contributed so much to the comfort and happiness of man, by raising his ideals to the advancement of civilization.

We do not speak of the "cultured world" using culture in the narrow sense in which the term is usually applied. That application is the direct antithesis of everything that science or mathematics would seek to accomplish. It is at best provincial, and, with almost no exception, it is artificial and rests on a foundation of untruth and pretence. The culture to which we refer is that development which results from the training and strengthening of all man's powers, mental and physical, with the consequent improvement or refinement of his mind, his morals, his tastes, and all his physical faculties, to the end that he may better fulfill his mission on earth by more freely discharging his duties to his fellow-creatures, to himself, and to his Creator.

Mabie reminds us that "culture is never a taking on from without, of some grace or skill or knowledge, it is always an unfolding from within into some new power, the flowering of some quality hitherto dormant, the absorption of some knowledge hitherto unappropriated. The essence of culture is not possession of information as one possesses an estate, but absorption of knowledge into one's nature so that it becomes bone of our bone and flesh of our flesh. It means the enrichment and expansion of the personality by the taking into ourselves of all that can nourish us from without. Its distinctive characteristic is not extent but quality of knowledge, not range but vitality of knowledge, not scope of activity but depth of life."

A surprisingly large part of all education which men get one from another is moral, not intellectual. Education as such, may consist merely in the acquisition of facts, the storing of them in the strong box of a brain instead of in the musty tomes of a library. Intellectual education demands that this acquired treasure of facts be put to work, that it shall unselfishly toil for others. It is this useful intellectual education which we call culture, not the counterfeit tawdry tinsel of fruitless knowledge. The education which reminds us that we are, after all, our brother's keeper, and that each act, each thought of ours, must somehow, someway work either for weal or woe upon some other creature, or group of creatures, placed by a common Father in our keeping.

Matthew Arnold has made the aim of culture not merely to render an intelligent being more intelligent, to improve our own capacities to the uttermost, but, "to make reason and the Kingdom of God, prevail." He makes culture an impelling force which stimulates the desire, not only to see things as they are, but rather, by the moral endeavor to know and understand more and more the universal order by which all created activity is regulated, and by this knowledge and understanding to conform to it ourselves, make others conform to it, and in this way to help to make the will of God prevail in us and around us.

The aim of true culture therefore, is not alone the securing of the best knowledge, the most accurate science, but to make them tell on human life and character, to the end that human nature in all its capacities may more closely approach perfection.

> "Unless above himself, he can
> Erect himself, how poor a thing is man."

Mathematics is not artificial. It is absolutely true, possessing beauties in its truths, and waging war ever and always, relentless war against untruth. It dwells not in the twilight zone of compromise, but in the noon-day glare of fact. The mind unfolds and develops when it reflects upon the subtle harmonies and the affinities of number and of magnitude. Its dormant qualities flower when it gradually solves a few "small simple postulates and a system of interesting theorems expanding into infinite and unexpected uses and applications." Mathematics is more than an organized knowledge of things, facts and events,

in their true relation and coordination, their antecedents and consequences. "No number of facts or aphorisms learned by heart makes a man a thinker, or does him much intellectual service." The elementary truths of number and space lie quite outside the region of contingency or controversy, and in consequence furnish ample and reliable material for the acquisition of habits of accurate thinking and deductive reasoning, so indispensable to success in all the pursuits of life, whether in material or spiritual activity. "Take a geometrical axiom, an elementary truth concerning the properties of space—two straight lines cannot enclose a space—or in arithmetic the elementary truth concerning the properties of number—to multiply by two numbers successively, is to multiply by their product, and we observe that the moment we state them, we perceive the necessary truth, there is no room for debate or difference of opinion; to understand either statement is to accept it. And so with all other fundamental axioms of mathematics. Whatever particular facts prove ultimately to be contained in these general or universal truths must be true. As far as we can be certain of anything we are certain of these."

St. Augustine appreciated the value of mathematical certitude, and made applications of it in his search for a knowledge of God and the soul. He employed the same plan of logical reasoning, and placed comparatively little worth on knowledge received through sense perception, unless followed up by careful investigation and reinforced by the intellect. "Wherefore it seems to me, that one could more easily sail on land, than learn geometry by means of the senses. If the knowledge of God and geometry were on a par, I would rejoice as much in knowing geometry, as I presume I shall rejoice once I know God. Now, however, in comparison with the knowledge of God, I so greatly contend, knowledge of geometry, that sometimes it seems to me, that if I knew Him as I see He can be known, I should quite forget my knowledge of geometry. Since indeed, in the presence of His love, the thought of geometry scarcely comes to my mind, and I am led to acknowledge that in its own field as far as the earth differs from heaven, so far do those true and definite truths differ from the intelligible majesty of God."

How often have not purely mathematical symbols, because of their symplicity, been used to point out abstract supernatural

truths to untrained minds. St. Augustine finds in the equilateral triangle and the square, the symbol of justice. The
circle is the typical figure for eternity, since it has neither a
beginning nor an end. The superimposed triangles, the star of
the House of David, led the Jews of the Old Law while the
chosen people lived in the light of God's blessings, and today it
is still their symbol of hope of the Messiah yet to come. Cardinal
Nikalous of Cusa compared the center of the circle to God as
the efficient cause of the universe, indivisible and simple.
''God's perfect simplicity does not consist merely in His indivisibility, but primarily, in the simultaneous plenitude of His
positive perfections of being.'' The locus of all points equally
distant from a common point is a circle, or in accordance with
the genetic definition, a circle is a figure formed by a point
moving in its plane at a constant distance from another point
in the same plane.

Among the characteristics of a cultured intellectuality I name
the defense of ideality, and the maintenance of spirituality. It
is objected, ideality and spirituality can have no place in mathematics. Cold, brutal, inelastic mathematics dealing with its
rules, its angles and its circles. Stop for but a moment and
consider, the very essence of the science, and it will be seen at
once that it is from its very nature abstract. Indeed its very
abstractness is usually of a higher order than the abstractedness of the logician. Thus we find imagination entering into
the citadel of truth, and certitude, and claiming the right
of intimate companionship with the multiplication table and the
propositions of Euclid. The whole study of geometry is an
imaginative study. The lines with which the mathematician
deals are not imperfect lines, drawn by the straightest pen, not
the finest gossamer web, woven by Queen Mab's tiniest spider,
but ideal lines which have length without breadth, and which
therefore, can exist only in the cultured imagination of the
highly trained mathematician. The sharpest blade produced in
Damascus and sweeping, with all but infinite swiftness, into the
most faintly resisting substance, would leave an angle too crude
to be dealt with by the mathematician; so he retires to the
quiet confines of his cultured mathematical mind, and there,
from the fabric from which the ordinary mortal weaves his
dreams, he constructs his angles whose sides are perfect, and

places them as his offering on the altar of absolute mathematical truth. No eye has ever seen or ever will see a circle or a square which complies with the definition of the circle or the square of the mathematician. The thing defined exists only in the mathematician's imagination, and every proposition in geometry involves the exercise of that faculty.

Most of us are forced to admit that the workings of our imaginations are bounded by the three dimensions of space in which we live and move—length, breadth and thickness. Out of them we model our mental conceptions, but not so the mathematician. He forms an imaginative conception of space, of four or five, or "n dimensions" and draws up laws which would govern his imaginative universe with absolute precision if its walls of mystic fiction were metamorphosed into the imperfect materials of fact. The idealist in those abstruse branches of science which deal with the problems of the ultimate construction of matter, and of the laws governing the forces which act upon it, arms himself not alone with his instruments of precision, his rules and his formulae, but finds them powerless without the aid of his mathematical imagination.

The astronomer climbs to his observatory and with his space-defying lenses peers into the hidden mysteries of a thousand worlds. With uncanny certainty he predicts phenomena to generations as yet unborn with a precision that confidently invites the closest scrutiny of his fellows, he weighs and measures the heavenly bodies, maps their courses, and calculates their velocity as they sweep through space. I am convinced that the lense of a Copernicus or Galileo would be worthless if hand and eye had not been directed by a truly cultured imaginative mind.

And so one might go through all the sciences, and find that those intellects that are productive of most valuable results in the realm of original research are strange combinations of the real and the unreal, the practical and the imaginative, each quality of mind, essential for the proper fulfillment of the other.

> "For the dreamer lives on forever,
> But the toiler dies in a day."

I have endeavored to show the close relationship existing between mathematics, by its very nature, and culture in its truest sense. That culture demands love of truth, that mathematics

depends essentially upon the attainment of truth, that true culture transcends the material things of life, that mathematics in its abstractions refuses to be bound by physical laws. It remains only to show that in its practical application mathematics remains a faithful servant to that idealism which points the way to higher and to better things, and brings man to a better knowledge and understanding of his Creator.

It may be said without fear of contradiction that there is no calling to which men have devoted themselves that has not received aid from applied mathematics. If then, mathematics has as its ultimate end the attainment of truth, it cannot but have been a faithful servant in the universality of its workings.

The architect, whether he follow any of the three great schools of the world, the Greek, Romanesque or Gothic, applies his mathematical rules to the straight line, the curve, and the angle, and behold, the Parthenon, the Coliseum, Rheims and St. Peter's spring into being. The Chaldeans, the greatest mathematicians of ancient times, in the beautiful tile construction and mosaics of the Alhambra, and the Mosque of Omar at Jerusalem, have given the modern world something better than tales of debauchery and bloodshed by which they were otherwise to be remembered. The mystery of the sphinx, the square beauty of the pyramids, and the scientific construction of their obelisks, have given to the world a sweeter dream of the Egypt of old, than the story of the power of the Pharaohs, or the corruption of Cleopatra. The roads of ancient Rome, over which rattled the chariot of all-conquering Caesar, as he led his troops to triumph, remain as a monument to his mathematical genius, though

　　"Imperial Caesar—dead and turned to clay,

　　May serve to stop a hole and keep the wind away."

If our museums are filled with collections of the sculptured arts of the ancients, and if the walls of the galleries and cathedrals of the world are rich with the colors of the masters of the brush, if bridges span our streams, if levies hold back the angry floods, if men speed across land and water, and if they have conquered the air, if messages of sorrow or rejoicing, of loss or gain, travel lightning-like through space, if contented labor pours its incense from a million forges to the heaven, then, indeed, has the science of mathematics justified its creation, for none of these successes could have been had not 2 plus 2 made

4, had not truth proved its might against doubt and ignorance and error.

Wherever ignorance is conquered, wherever error is overcome, man is brought into more healthful relationship with his fellow-man, and into closer communion with his Creator. St. Augustine has said, "That is the true perfection of a man, to find out his own imperfections." No science which unduly exalted a creature would truly serve him. Mathematics pointing always to absolute truth as its ultimate goal, points also to the limitations of all human endeavor, to the frailty of finite intelligence, to man's inability to travel, in the workings of any of his faculties beyond the narrow confines set for him by the Master who brought him out of nothingness.

Thus mathematics by developing the higher intellectual faculties of the individual, making him the unselfish servant of his brother, and bringing him into closer knowledge of his Maker, fulfills the requirement of a cultural science. If pride closed the gates of heaven against the vanity of Lucifer and man in intellectual pride attempts to follow him, he must first leave by the roadside that faithful companion in dull drab raiment, the burden of whose song as he trudges along is ever "2 plus 2 makes 4," "Truth is mighty and must prevail."